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A New Method for Estimation of Ex ante Equity Risk Premiums

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Julkaisun nimike Menetelmä ex ante -riskilisän estimoimiseksi		

Tiivistelmä

Osakkeiden ex-ante riskilisää ovat kirjallisuudessa käsitelleet mm. Kocherlakota (1996), Claus and Thomas (2001) and Fama and French (2002, 2006). Mallien soveltamisessa käytäntöön ei kuitenkaan ole saavutettu konsensusta kuten Welch (2000) toteaa. Artikkelin tarkoitus on käsitellä Welch'in (2000) esittelemää ongelmaa kehittämällä sovellus osakkeen riskilisän laskemiseksi. Menetelmä määrittelee riskilisän osakkeen tulosennusteiden, osakkeen kurssin ja riskittömän korkokannan avulla. Menetelmä soveltuu työkaluksi mm. osakeanalyysejä tekeville. Kehitetty menetelmä mahdollistaa erilaiset vertailut, kuten esimerkiksi maiden ja toimialojen väliset vertailut.

JEL-luokitus: G12, G31

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Abstract

Even though the estimation of the expected equity risk premium has gained some attention in recent literature (see e.g. Kocherlakota 1996, Claus and Thomas 2001 and Fama and French 2002, 2006), there is no consensus on how the models used in the estimation should be applied in practice, as Welch (2000) states. The purpose of this paper is to tackle the problem stated by Welch (2000) by developing an application to estimate the risk premium of a stock or whole stock market by exploiting the risk measure developed in the paper. The method can define the risk premium as a function of earnings forecasts, the current price of the equity and risk free rate of return. From a practical point of view, the method developed is especially suitable as a tool for analysts in their everyday work. It is also appropriate for different kinds of comparisons since the risk premium measure has the same meaning, for example, across countries and industries.

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Keywords
risk premium, equity valuation

Contents

1. INTRODUCTION

Despite the fact that the central role of the equity risk premium in financial economics has been understood for a long time among academics and practitioners, the estimation of it is a fairly new phenomenon, as Goetzmann and Ibbotson (2005) state. The first attempts to investigate the risk premium concentrated on the measurement of the historical risk premium. Those papers, including the seminal work by Mehra and Prescott (1985), observed the so-called equity premium puzzle, i.e. the historical risk premium is higher than predicted based on theoretical economic models. After the development of the CAPM, which relates the risk of a security and the expected return but requires the expected market risk premium as an input, the interest in expected, i.e. ex ante, risk premium has increased.

However, the practical application of CAPM is rather problematic. In principle, it is possible to estimate risk premium for the total equity market using e.g. valuation models, even if this is rather cumbersome. Probably an even more severe problem is the estimation of the beta, since usually estimation relies on historical data. The problem is that the beta changes over time as risk of the stocks changes. This problem has been observed in several studies and Fama and French (1992, 1993, 2004 and 2006) and Fernández (2004) for example find that the beta has hardly any predictive power with respect to stock returns. Actually, Fama and French (2004) state that ‘the failure of the CAPM in empirical tests implies that most applications of the model are invalid’. Given this empirical evidence, the practical application of the CAPM is very difficult.

In addition to the CAPM, the current literature suggests several possible methods for the estimation of the expected risk premium of equity. The simplest way is to calculate the historical risk premium and assume that history will repeat itself. For example, Ibbotson and Chen (2003) find that the historical risk premium was around six percent during the period 1926–2000. Probably a more realistic approach would be to make a survey study where a question about the expected risk premium is asked from the professionals applying the risk premium in practice. One such is a survey by Graham and Harvey (2005), which finds that the CFO’s mean expected market risk premium was 3.7 % in June 2005. A similar approach can be used to assess the risk premium of a particular stock.

An alternative way to measure equity risk premium is to derive it from the market prices of stocks. If the stock market is efficient, stock prices reflect the time value of money, including the risk premium, correctly. Thus, market prices contain investors’ expectations of the risk premium. If the market is inefficient, the measure is still the market expectation but differs from the unobservable correct risk premium.

In this approach, some equity valuation model is used to obtain the risk premium that can be regarded as the market's consensus view about the risk premium. The background of these methods is in the models that exploit market and financial data, such as growth forecasts, dividends, earnings and the required rate of return to value stocks. Alternatively, these models can be used to estimate the expected risk premium by plugging in stock price as a value and solving the required rate of return, which, in this case, is the market expectation of return. Probably the most famous of these models is Gordon's (1962) model. It defines the price of a stock as a function of current dividend payment, constant dividend growth rate and required rate of return. Alternatively, the risk premium of a stock is obtained by plugging in the market price of a stock and solving the required rate of return. For example, Claus and Thomas (2001) and Fama and French (2002) use this type of approach and find that the expected risk premium is around 3 % in the U.S.[1] However, there is no consensus on how those models should be used and applied in practice to obtain the risk premium for a market or a stock, as Welch (2000) states.

The purpose of this paper is to tackle the problem stated by Welch (2000) by developing an application to estimate the risk premium of a stock by exploiting an explicit risk measure developed in the paper. We use very general mathematics, and the method makes it possible to use any expected earnings distribution, not only the exponential one. The method can be applied in practice by using earnings forecasts, the current price of the equity and the risk free rate of return. Given that earnings forecasts and stock prices are nowadays widely available, the method can be applied fairly easily to calculate the market's expected risk premium for an aggregate equity market or for a particular stock. This enables us, for example, to analyze changes of equity risk premium in real time, which is an important benefit for analysis.

The remainder of the paper is organized as follows. In the next section, the theoretical background is described. In Section 3, the Earn Back Period measure is derived. The measure of the equity risk premium is developed in Section 4. Section 5 assesses the properties of the measure developed in the previous section. Section 6 provides some practical examples. Section 7 concludes the paper.

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[1] Luoma, Sahlström and Ruuhela (2006) have presented an algorithm for the calculation of ex-ante equity risk premium. The method is based on the pay back method and is limited to stocks with a constant growth rate of earnings per share.

2. THEORETICAL BACKGROUND

In capital budgeting, a commonly used method to evaluate real investments is the payback period. For example, Graham and Harvey (2001) survey 392 CFOs of U.S. companies and find that the payback period is among the most used methods. In particular, it is the most used method with NPV and IRR among the smallest companies. This is rather surprising, because finance textbooks usually warn about the shortcomings of the method. These shortcomings are that the payback ignores the time value of the money and cash flows after the payback period. However, McDonald (1998) notes, using option valuation theory, that the payback period can approximate optimal decision rules with risky investments. Similar results, using a different approach, have been reported by Boyle and Guthrie (2006). Moreover, the general use of payback period is probably due to its simple calculation and that it emphasizes the liquidity aspect of the investment which is consistent with the view that cash flows are more uncertain in the distant future.

We exploit the payback concept to evaluate stocks, i.e. we use expected earnings to define how many years it takes for a stock to earn back its share price. Even though this method still has the shortcoming of not taking cash flows (or earnings) after the payback period into account, it is not such a severe problem according to McDonald (1998) and Boyle and Guthrie (2006) since those cash flows are very uncertain. Moreover, the second shortcoming, time value of money, is taken into account since the stock market values shares so that the stock price reflects the time value of money.

In the case of a risky project and a risk free project, the risky project should have a shorter payback period to compensate for the higher risk. Generally speaking, a riskier project should have a shorter payback period. Similarly, in the case of two stocks, the riskier should have a shorter payback period. Using this basic logic and the fact that differences in growth rates are captured by earnings forecast, we show that the expected payback of stocks can be used as a risk measure and thus can be used to evaluate stocks.

We call the time needed to earn the whole stock price back, i.e. payback period, the *Earn Back Period* (*EBP*). It can be calculated using the current market price of stock and earnings forecasts. Thus the *EBP* is defined to be the expected number of years that a firm needs to earn an amount equal to the stock price.

To allow different types of applications and to keep our development of the concept general, we use continuous mathematics. Finally, a discrete case is included as a special case. Hence we will use an earnings inflow function $f(t)$. It gives the earnings inflow rate and has the property

(1) $$\int_{n-1}^{n} f(t)dt = E_n$$

where E_n are earnings per share in the year *n*.

Formally *EBP* is defined by the equation

(2) $$\int_{0}^{EBP} f(t)dt = P$$

where

EBP = Earn Back Period,
f(t) = earnings inflow,
P = price of the stock at the beginning of the first year, i.e. current stock price.

EBP is a risk measure for the stock; riskier stocks have shorter Earn Bach Periods. The final risk measure for the stock we get by transformation

(3) $$RM_i = EBP_f - EBP_i,$$

where

RM_i = Risk Measure for a company i,
EBP_f = Earn Back Period for risk free security,
EBP_i = Earn Back Period for a security at hand.

A common definition of the risk premium, i.e. that the risk premium is the difference between an asset's expected rate of return and the risk free rate of return is an analogue with Equation (3). However, Equation (3) gives risk premium in years instead of per cents. RM_i equals zero for a risk free security and increases as risk increases. Hence *RM* has better properties as a risk measure than *EBP*. RM_i according to Equation (3), is a monotonic transformation of EBP_i. Thus both measures give the same order of riskiness for stocks. For the sake of a lucid interpretation we prefer *EBP* over *RM*, and will use only *EBP* in remaining of the paper.

3. EARN BACK PERIOD

When forecasting future earnings, it is common to estimate the few first years $E_1,...,E_n$ individually, because they may be exceptional and not follow any certain growth rate. Earnings after the period *n* are usually estimated by some growth function, mostly by an exponential growth function. Thus *EBP* may be calculated by an equation

$$(4) \qquad \sum_{1}^{n} E_t + \int_{n}^{EBP} f(t)dt = P$$

Under the assumption of exponential growth, the second term of Equation (4) is defined as follows:

$$(5) \qquad f(t) = ae^{bt}$$

For practical usability, a constant percentage growth rate, *b*, is probably the most suitable. If some other form than exponential growth rate function is needed, then $f(t)$ in Equation (4) is replaced by the function in question. Hence the method is completely general.

The solution of *EBP* from Equation (4) using the growth rate function (5) is

$$(6) \qquad EBP = n + \frac{1}{b}\ln[\frac{e^b - 1}{E_{n+1}}(P - \sum_{1}^{n} E_t) + 1]$$

Year *n+1* is usually called a normalized year by analysts, because E_{n+1} and estimated growth rate *b* are used for the calculation of expected earnings for years from the year *n+1* and to infinity. The years *1,2,…,n* are usually estimated separately.

It is to be noted that with this formula (6) we can already rank stocks according to their risk. The smaller the *EBP*, the riskier the stock in the market’s view.

We will now consider Earn Back Period in three important special cases:
1. earnings are constant over the years
2. earnings grow with a constant percentage rate and
3. a security is risk free.

The first point means a constant earnings inflow, i.e. $b = 0$ or $f(t) = a$. From Equation (2) we have

(7) $$\int_0^{EBP} a\,dt = P$$

Simple calculation gives

$$EBP = \frac{P}{a}$$

But according to Equation (1) we get

(8) $$E = \int_{n-1}^{n} a\,dt = a$$

Hence in the case with the constant earnings we get a well known implementation

(9) $$EBP = \frac{P}{E}$$

Earn Back Period is simply the *P/E* ratio for firms with the constant earnings. The *P/E* ratio is probably the most used method for equity valuation and comparison on the stock market. It is also well known that there are difficulties in using the *P/E* ratio for growing companies since the ratio does not take into account different growth rates.

The second special case analyzed is that the growth rate of earnings is positive and constant. This leads to the following growth function, as we have already pointed out above:

(10) $$f(t) = ae^{bt}$$

The parameter b in the earnings function is a growth parameter (compound growth rate) and the parameter a is a size parameter and it is dependent on the company's size. Again using Equations (2) and (10) we get

(11) $$EBP = \frac{\ln(1 + \frac{b}{a} P)}{b}$$

We want to exclude the unillustrative size parameter *a* by replacing it with the solution from

$$E_1 = \int_o^1 ae^{bt} dt$$

and dropping a subscript.

(12) $$EBP = \frac{\ln[1 + (e^b - 1)\frac{P}{E}]}{b}$$

We can also get this equation as a special case from Equation (6) by defining n = 0. In that case the sum in the equation disappears. Furthermore, E_{n+1} is substituted by E.[2]

So far we have consistently used continuous mathematics and its notations. In the following a discrete interest rate and discrete growth rate are used because they are used in practical applications. The relationship between the compound interest (or growth) rate b and the respective discrete interest (or growth) rate r is as follows,

(13) $$b = \ln(1 + r) \text{ or } e^b = 1 + r$$

These equations will be used when continuous mathematics is applied in practice. They will be used in the third special *EBP* case. In point of fact, the third case is included in the second case, because interest payments are reinvested every year. Using equation

(14) $$E_1 = E = r_f P ,$$

where r_f is (usual or discrete) interest rate for risk free asset, and using the latter of Equations (13) for risk free interest rate and Equation (12) we get *Earn Back Period* for the risk free security

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[2] In Luoma and Ruuhela (2001) a discrete analogue for Equation (12) is presented.

(15) $$EBP_f = \frac{\ln 2}{b_f}$$

where b_f is a compound risk free interest rate. The same in discrete form is as follows:

(16) $$EBP_f = \frac{\ln 2}{\ln(1 + r_f)}$$

4. EQUITY RISK PREMIUM

A stock's risk premium can be derived by utilizing the property of the *EBP* that differences in *EBP* are due to differences in risk. To obtain a stock's risk premium we can solve the *EBP* for a stock and calculate, according to the definition of the risk premium (Δr), how much must be added to r_f in Equation (16) so that EBP_f becomes equal to the *EBP* of the stock in question, i.e. we have to solve an equation with respect to Δr:

(17) $$EBP = \frac{\ln 2}{\ln(1 + r_f + \Delta r)}$$

The solution for the risk premium is

(18) $$\Delta r = \exp(\frac{\ln 2}{EBP}) - 1 - r_f$$

The equation shows that the risk premium decreases steadily as the *EBP* increases, i.e. the risk premium is a monotonic transformation of the *EBP*. Equation (6) is a general formula to calculate *EBP*. Thus, we can calculate equity risk premium with Equation (18) in every case when it is possible to calculate Earn Back Period *(EBP)*.

5. ASSESSMENT OF THE MODEL

In the future, the validity and reliability of our measure of the equity risk premium should be empirically tested. However, we can make some assessments from a theoretical point of view. In this way we can detect possible inconsistencies and point out logical consistencies.

To consider consistency we want to see if the impacts of different factors in the model are logical. We solve Equation (6) with respect to the price using Equation (17) and obtain

(19) $$P = \frac{E_{n+1}}{e^b - 1}\{\exp[b(\frac{\ln 2}{\ln(1 + r_f + \Delta r)} - n)] - 1\} + \sum_1^n E_t$$

Contributing factors for the price are

$E_1, ..., E_n, E_{n+1}$ = earnings from *n+1* first forecasting periods,
b = growth rate of the earnings $E_{n+1}, E_{n+2}, ...$
r_f = risk free interest rate and
Δr = equity risk premium.

Based on basic theories in finance the following properties have to hold for our measure:

- positive relationship between earnings and price
- negative relationship between the risk free rate of interest and price
- negative relationship between the risk premium and price
- positive relationship between the growth rate and price.

According to Equation (19) all claims except the last one are apparently correct. However, it can be shown that the increase of growth rate of earnings also increases the price of stock on all practical levels of earnings growth rate[3]. When considering the impact of the variables on the price we should not forget that, in practice, variables also have indirect impacts. For example, if a current earnings

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[3] If growth rate of earnings rise, so does price except under conditions when the sum of risk free interest rate and equity risk premium equals 100 per cent or more, which case is in practice infeasible. We thank Dr. Matti Laaksonen of the University of Vaasa for the mathematical proof.

variable increases, other variables, e.g. earnings growth rate may also change, having influence on the price. Thus, if we consider one variable at a time and only a direct impact we get just a limit for the price change. For example, suppose that an interest level decreases. It means that the price increases. Substituting a new interest rate r_f into Equation (19) but keeping other variables unchanged we obtain the minimum value for the new price.

6. AN EXAMPLE OF THE APPLICATION

As an example we take Motorola. Its consensus estimates from September 12, 2006 are obtained from the data file of Thomson Financial. They are as follows.

Year	*EPS*	*Div/Share*	*ROE%*
2007	1.33	0.17	18.29
2008	1.52	0.17	16.17
2009	1.80		

where

EPS = earnings per share,
Div/Share = dividend per share,
ROE% = return on equity.

The stock price was $23.97 and the risk free interest rate used 5 %. Using these numbers we estimated growth rate with the equation

(20) $$(EPS_{08} - Div_{08})ROE_{08} \,/\, EPS_{08}$$

and obtained 14.4%. Using the estimated growth rate we can estimate EPS for every year in the future starting from 2009. Moreover, the Earn Back Period can be calculated with Equation (6), using Transformation (13). Finally Equation (18) yields the risk premium 2.69 %. It is presumably lower than the risk premium for the whole market. In practice, changes in risk premiums over time are especially interesting.

In further analysis, we can use Equation (19) to estimate the minimum impact on price when some of the factors of the risk premium change. In the following, there are some examples of changes in price when a factor changes, ceteris paribus.

	Change of the factor	Change of the price
Interest rate	0.25 percentage units	-5.3%
Growth rate	1 percentage units	2.9%
EPS year 07	10 per cent	0.6%
EPS year 08	10 per cent	0.6%
EPS year 09	10 per cent	8.8%
Risk premium	0.5 percentage units	10.0%

Based on the above, we can make some interesting observations. Minimum change in price is a decrease of 5.3 % for an increase of twenty five points in the interest rate. Thus, it is no wonder that investors, not only Motorola owners, follow changes in interest rates very closely. The most sensitive of EPS values is the last one, because it is the basis for the calculation for all future EPS estimates. Risk premium and interest rate changes have the same (and fairly big) influence on price. This means that overall market sentiment is an important factor in determining the price.

7. CONCLUSIONS

Even though the estimation of the expected equity risk premium has gained some attention in recent literature (see e.g. Kocherlakota 1996, Claus and Thomas 2001 and Fama and French 2002), there is no consensus on how the models used in the estimation should be applied in practice, as Welch (2000) states. The purpose of this paper is to address the problem stated by Welch (2000). Thus in paper, we develop an application to estimate the risk premium of a stock or whole stock market by exploiting the risk measure developed in this paper. The method can define the risk premium as a function of earnings forecasts, the current price of the equity and risk free rate of return. Thus it is easy to apply in practice to obtain the ex ante risk premium.

From the practical point of view, the method developed is especially suitable as a tool for analysts in their everyday work. The method developed can be used to make different kinds of comparisons since the risk premium measure has the same meaning, for example, across countries and industries. Investors can use the risk premium to optimize international diversification by calculating risk premiums across countries and comparing their riskiness. This means that those countries that give the highest risk premium per unit of risk are the most promising. A similar type of analysis can be conducted to compare industries and single stocks. As future research, the performance of the risk premium measure should be investigated in a context similar to that in which the beta of CAPM has been tested, for example, by Fama and French (1992) and Lakonishok, Shleifer and Vishny (1994).

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